Statistics:
Uniform Distribution (Continuous)

The uniform distribution (continuous) is one of the simplest probability distributions in statistics. It is a continuous distribution, this means that it takes values within a specified range, e.g. between 0 and 1.

The probability density function for a uniform distribution taking values in the range a to b is:

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

Example

You arrive into a building and are about to take an elevator to the your floor. Once you call the elevator, it will take between 0 and 40 seconds to arrive to you. We will assume that the elevator arrives uniformly between 0 and 40 seconds after you press the button. In this case $a = 0$ and $b = 40$.

Calculating Probabilities

Remember, from any continuous probability density function we can calculate probabilities by using integration.

$$\mathrm{P}(c \leq x \leq d) = \int_c^d f(x)\,dx = \int_c^d \frac{1}{b-a}\,dx = \frac{d-c}{b-a}$$

In our example, to calculate the probability that elevator takes less than 15 seconds to arrive we set $d = 15$ and $c = 0$. The correct probability is $\frac{15-0}{40-0} = \frac{15}{40}$.

Expected Value

The expected value of a uniform distribution is:

$$\mathrm{E(X)} = \int_a^b x f(x)\,dx = \int_a^b \frac{x}{b-a}\,dx = \frac{b-a}{2}$$

In our example, the expected value is $\frac{40-0}{2} = 20$ seconds.

Variance

The variance of a uniform distribution is:

$$\begin{aligned} \mathrm{Var(X)} &= \mathrm{E(X^2)} - \mathrm{E^2(X)} \\ &= \int_a^b \frac{x^2}{b-a}\,dx - \left(\frac{b-a}{2}\right)^2 = \frac{(b-a)^2}{12} \end{aligned}$$

In our example, the variance is $\frac{(40-0)^2}{12} = \frac{400}{3}$

Standard Uniform Distribution

The standard uniform distribution is where $a = 0$ and $b = 1$ and is common in statistics, especially for random number generation. Its expected value is $\frac{1}{2}$ and variance is $\frac{1}{12}$

Statistics:
Uniform Distribution (Discrete)

The uniform distribution (discrete) is one of the simplest probability distributions in statistics. It is a discrete distribution, this means that it takes a finite set of possible, e.g. 1, 2, 3, 4, 5 and 6.

The probability mass function for a uniform distribution taking one of n possible values from the set $A = (x_1, .., x_n)$ is:

$$f(x) = \begin{cases} \frac{1}{n} & \text{if } x \in A \\ 0 & \text{otherwise} \end{cases}$$

Example

DICE??

Calculating Probabilities

Remember, from any discrete probability mass function we can calculate probabilities by using a summation.

$$\mathrm{P}(x_c \leq X \leq x_d) = \sum_{i=c}^{d} f(x_i) = \sum_{i=c}^{d} \frac{1}{n}$$

In our example, to calculate the probability that the dice lands on 2 or 3 we set $d = 3$ and $c = 2$. The correct probability is $\frac{1}{6} + \frac{1}{6} = \frac{2}{6}$.

Expected Value

The expected value of a uniform distribution is:

$$\mathrm{E(X)} = \sum_{i=1}^{n} x_i f(x_i) = \sum_{i=1}^{n} \frac{x_i}{n} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{x_1 + x_n}{2}$$

In our example, the expected value is $\frac{1+2+3+4+5+6}{6} = \frac{1+6}{2} = 3.5$.

Variance

The variance of a uniform distribution is:

$$\mathrm{Var(X)} = \frac{(b - a + 1)^2 - 1}{12}$$

In our example, the variance is $\frac{(6-1+1)^2-1}{12} = \frac{35}{12} = 2.9$

Standard Uniform Distribution

The standard uniform distribution is where $a = 0$ and $b = 1$ and is common in statistics, especially for random number generation. Its expected value is $\frac{1}{2}$ and variance is $\frac{1}{12}$